Discretizing υ-Andromedæ planetary system

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Abstract

Starting from a previously stated hypothesis concerning the discretization of the orbits for periodic celestial motions, the mass of υ Andromedæ and the periods of its b- and d- companions are estimated using the accurately known period of the massive c-companion. Moreover, the periods and the orbital major semiaxes of other hypotetically existing and not yet discovered smaller planets of υ Andromedæ are guessed.

1 Introduction

In a previous work [1] the hypothesis was tested that periodic celestial motions comply with orbital discretization rules similar to those given by Bohr-Sommerfeld for the Old Quantum Mechanics. In the case of a single planet gravitating in the field of a star of mass M the discretization rules can be written

$$\oint v_i \, dx_i = n_i \, 2\pi \frac{G \, M}{\alpha_g c} \qquad (i = 1, 2, 3) \; , \tag{1}$$

where G is the gravitational constant, c the light speed and α_g a dimensionless gravitational structure constant to be determined on the basis of the observational data. From (1) the following relations concerning the orbital

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major semiaxis a_n and the corresponding orbital periods P_n can be easily deduced

$$\begin{cases} a_n &= n^2\, a_1\ ; \qquad [a_1 = \frac{GM}{\alpha_g^2 c^2}] \\ P_n &= n^3\, P_1\ ; \qquad [P_1 = 2\pi \frac{GM}{\alpha_g^3 c^3}]\ . \end{cases} \tag{2}$$

Applying this scheme to the solar sistem we obtained (see [1]) the following selection for the quantum numbers n: Mercury (3), Venus (4), Earth (5), Mars (6), Jupiter (11) , Saturn (15), Uranus (21), Neptune (26), Pluto (30), and the numerical specifications

$$\begin{cases} \frac{1}{\alpha_g^{\odot}} &= 2086 \pm 14 \\ \\ a_1^{\odot} &= 0.04297\ a.u. \\ \\ P_1^{\odot} &= 3.269\ d\ , \end{cases}$$

where the symbol $^{\odot}$ refers to solar system related quantities. The fit was performed both giving equal statistical weights to all planets and taking into greater account Jupiter, which, owing to its mass, is reasonably the less perturbed (almost isolated) planet.

Note that on account of Eqs. (2) for every celestial planetary system (of almost non interacting planets) the equation holds

$$\frac{P_n}{P_n^{\odot}} = \frac{M}{M^{\odot}}, \tag{3}$$

so that one can use the careful measurements referring to the solar system to perform adequate comparisons.

On the basis of the same discretization pattern (1), the fundamental value for the dynamical spin

$$J = \frac{GM^2}{2\alpha_g^{\odot} c}$$

was found, which surprisingly holds well for all celestial systems, from planets to superclusters, on about twenty orders of mass magnitude, in agreement with the Brosche-Wesson law ([2] [3] [4] [5]). Thus, on account of this result, we provisionally identified the $\alpha_g^{\odot}$ with the gravitational structure constant α_g.

2 The extrasolar planets

In a subsequent work ([6]), the previously stated discretization rules were applied to the recently discovered extrasolar planets (in fact about twenty star-planet pairs). The comparison with experimental data gaves generally fair results, mainly for planets with small orbital number ($n \leq 2$). In fact, for most of such systems, only the planet period was known with sufficient accuracy, whereas the star mass was usually very roughly estimated from the star type: of course, this fact produces some inaccuracy on the estimates, mainly for large orbital number. When $n = 1$, one can compare the quantity

$$P_1^{(est)} = P_1^{\odot} \frac{M}{M^{\odot}} \tag{4}$$

with the observed value $(P_1)^{(obs)}$. In the following table we report some comparisons obtained using the previous equations and uptaded observational values ([7]) :

Star	Star type	$P^{(obs)}$ $[d]$	$a_1[a.u.]$	$P^{(est)}$ $[d]$
HD 187123	G5	3.097	0.042	3.19
tau Bootis	F6IV	3.313	0.046	3.50
HD 75289	G0V	3.508	0.047	3.58
51 Peg	G2IVA	4.231	0.051	3.88

Note that even for some planet with $n = 2$ (for instance,HD 192263, see [8]) one would obtain a fair agreement ($\mathrm{P}^{(est)} = 22.8$ $[d]$ *vs* $P^{(obs)} = 23.9$ $[d]$) If we recall that the reported major semi-axes have been calcolated using the (roughly estimated) star masses, we can note that all the examined planets satisfactorily follow our rule. The only disagreement would occur for the star-companion system HD 217107 ($(P_1)_{est} = 5.32$ d *vs* $P^{(obs)} = 7.11$ $[d]$).

3 The υ Andromedæ planetary system

In the last year, one of the previously discovered star-companion pairs, namely υ-Andromeda, has shown itself to be a real planetary system, with three planets, simply named b, c, and d (see [9]). Planet b, declared as very similar to Jupiter, is about 0.059 $a.u.$ from the star, and its period is 4.617 d. The mass of c-planet is about twice as the b-planet mass, and its orbital

semiaxis and period are respectively 0.83 $a.u.$ and 241.2 d. The third planet, at a distance 2.5 $a.u.$ from υ Andromedæ, has a mass about four times that of the first and a period is 1267 d .

We recall that, for the solar system, a simple fit based only on the almost unperturbed Jupiter (with $n = 11$) gave reliable results for the whole solar system. Likewise, we take the massive c-companion, whose period ($P = 241.2\ d$) is known with some accuracy, as the reference planet for the υ Andromedæ system and consistently assign $n = 4$ to its orbit. Then, we can immediately calculate the υ Andromedæ mass $M^{\upsilon A} = 1.15\ M^{\odot}$ (Gray [10] estimated $M^{\upsilon A} = 1.20\ M^{\odot}$, whereas Ford et al. [11] declared a value $M^{\upsilon A} = 1.34\ M^{\odot}$). Moreover, we simply get the period $P_1 = 3.76\ d$ for the b-companion and the period $P_7 = 1289\ d$ for the d-companion, both values being highly consistent with the observed values.

At this point, we can give a table with the guessed values of the periods and the major semiaxes of the υ Andromedæ planetary system, including the orbits of hypotetically existing, though not yet observed, smaller companions:

n	$P_n = n^3 P_1\ [d]$	$a_n = n^2 a_1\ [a.u.]$
1 (observed)	3.76	0.05
2	30.1	0.20
3	101	0.44
4 (observed)	241	0.79
5	470	1.23
6	812	1.78
7 (observed)	1289	2.42
8	1985	2.75
9	2711	3.46
10	3760	4.30

4 Conclusions

Assuming that simple discretization rules (see [1]) hold for stable periodic celestial motions, one is able in particular to calculate and compare the characteristic periods P_n of newly discovered planetary systems of a star of mass M using the relation

$$P_n = n^3 P_1^{\odot} \frac{M}{M^{\odot}} \ , \tag{5}$$

where $P_1^{\odot} = 3.27\ d$ is a reference period related to the solar system, $M^{\odot}$ is the solar mass, and n an integer number.

However, the reasons why the mentioned discretization rules hold, very satisfactorily or almost approximately, for a large group of periodic celestial motions are not at all clear at present. In our opinion, Nelson's stochastic mechanics ([12], [13]) applied to the protoplanetary matter using the value $D = GM/2\alpha_g c$ for the diffusion coefficient([1]), or fractal space-time schemes ([14], [15]) are today the best candidates to explain the effectiveness of our simple discretization rules. Nevertheless, we suspect that equivalent results could be obtained by looking for stable attracting orbits, in the sense of the chaotic dynamics, for the strongly nonlinear gravitational motions.

We explicitely note that our mechanical scheme does not agree with the recently proposed theories of the planet migration (see for instance [16]).

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